Income Distribution and Demand-induced Innovations

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Abstract

We utilize Schmookler's (1966) concept of demand-induced invention to study the relationship between income distribution and long-run growth. When innovations are demand-induced, the distribution of income becomes a crucial determinant for the market size and the prices of new products.

The inequality-growth relation turns out to be ambiguous and depends on the particular form of inequality. A higher concentration of incomes among a smaller group of consumers reduces the *size of the market* and depresses the incentive to innovate. A larger income gap between rich and poor consumers (holding group sizes constant), however, allows innovators to charge *higher prices* which fosters innovation and growth.

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”Does man simply invent what he can, so that the inventions he makes in any period are essentially those which became possible in the previous period? Or is it to man’s wants with their different and changing intensities, and to economic phenomena associated with their satisfaction, that one must primarily look for the explanation? In short, *are inventions mainly knowledge-induced or demand-induced?”*

Schmookler, Jacob (1966), Invention and Economic Growth, p.12.

1 Introduction

Theories of innovation and growth have been concerned with increases in the variety, quality, and volume of aggregate output either because of increased technical knowledge and/or because of (physical or human) capital accumulation. In these theories the supply of production factors, and determinants of their productivity, affect the volume and structure of aggregate output. The nature of human wants plays a subordinate role. Each new invention creates its own demand.

In his seminal book ”Invention and Economic Growth,” Schmookler (1966) forcefully argues against such a one-sided view and emphasizes the importance of ”demand-induced inventions”. He defines an invention as a new combination of both pre-existing knowledge *and the satisfaction of some want*. Not only the specific technological knowledge but also a sufficiently urgent want is necessary for an invention. In Schmookler’s (1966) words: ”Without wants no problems would exist. Without knowledge they could not be solved.”

Were all (unsatisfied) wants equally urgent, the notion of ”demand-induced inventions” would be meaningless. It would not matter what consumers want, it would only matter what producers know; innovations would be knowledge-induced. Hence the notion of ”demand-induced inventions” requires heterogenous wants. Whether a particular invention is made, does not only depend on whether it is technology feasible, but also on whether consumers want it badly enough.

In this paper we elaborate on Schmookler’s idea of demand-induced invention to show that the innovation process is inherently affected by the distribution of income. In the model developed below, we assume that households differ only in their income. They are identical in all other respects. There are infinitely many wants. The urgency of a particular want is determined by the utility a consumer derives from its satisfaction. Satisfaction of more urgent wants yields higher utility, whereas satisfying less urgent wants yields lower utility. This implies a ranking to which we refer as the "*hierarchy of wants*".

The distribution of income matters because richer consumers are able to satisfy more wants than poorer consumers. As new products satisfy less urgent wants, only rich consumers may have sufficiently high willingnesses to pay for new goods. Poor consumers may not be able to afford them. This simple fact implies that the distribution of income affects the value of an innovation and hence the attractiveness to pursue R&D projects.

The steady-state growth path in our model is characterized by the following features. First, innovations in the model are both "knowledge-induced" *and* "demand-induced". They are "knowledge-induced" because new innovations enlarge the knowledge base in the whole economy and make the satisfaction of new wants possible in the first place. They are "demand-induced" because consumers' preferences determine the direction of economic development. Our model studies Schmookler's (1966) introductory question by emphasizing *the level of a consumer's income* as "the main economic phenomenon associated with the wants' satisfaction." Clearly, when consumers earn unequal incomes, the intensity of wants *differs* across consumers because richer individuals have a higher willingnesses to pay for the same wants. Similarly the intensity of wants *changes* over time, because incomes and the willingness to pay for the various products increases.

Second, our framework sheds new light on the relationship between the distribution of income and economic growth. It highlights the potential importance of a channel that has previously been largely neglected in the literature. In our framework income distribution affects the evolution of prices and market sizes for new products. It turns out that changes in income inequality have two opposite effects. On the one hand, there is a *market size effect* by which less inequality – due to a lower concentration of income among a smaller group of individuals – fosters the incentive to innovate. If this effect dominates, inequality is harmful for growth. On the other hand, there is a *price effect* by which less inequality – due to a smaller income gap between rich and poor consumers – depresses the incentive to innovate by lowering the willingness to pay for new goods of the rich consumers. If this effect dominates, inequality is beneficial for growth.

Third, in our framework of income distribution and demand-induced innovations, the distribution of income determines the life cycle for a new product. Goods are initially luxuries purchased only by the rich and finally become conveniences, also affordable by the poor. Hence the distribution of income does not only affect prices and market size in the initial phase of the product cycle but determines the entire life cycle of a product.

Fourth, the equilibrium will typically be characterized by *exclusion of the poor from some markets*. Innovators face the choice between selling to rich consumers at high prices and to the whole market at low prices. Whenever the former strategy is more profitable the poor will not be able to afford the particular good. By "market exclusion" we mean that poor consumers have a willingness to pay that is above the marginal cost of production. They are excluded because innovating firms have market power and raise their price far above the marginal cost.

The paper is organized as follows. Section 2 provides a survey of the related literature. In Section 3 we present the specification of the demand-side of our model and Section 4 discusses the static equilibrium in detail. Section 5 describes the properties of the balanced growth path. Section 6 discusses possible other equilibria and Section 7 presents a comprehensive graphical representation of the dynamic equilibrium. In Section 8 we analyze the relationship between inequality and growth. In Section 9 we look at the robustness of our results with respect to the basic assumptions. Section 10 concludes.

2 Related literature

At a general level, our paper is related to models that emphasize the importance of market size for the incentive to innovate which is central in R&D based growth models (Aghion and Howitt, 1992, Grossman and Helpman, 1991, Romer, 1990, Segerstrom et al, 1990). However, in these models unsatisfied wants are all alike, so the issue of demand-induced inventions does not arise in any interesting way. Furthermore, these models typically assume homothetic preferences, so that income distribution does not have any role for the market demand functions of innovating firms.

A further important related literature is concerned with "directed technical change" (Kennedy, 1964, Acemoglu, 1998, 2002, 2003). These papers put emphasis on the incentives to adopt particular technologies and the consequences of these technology choices for the distribution of income among factors of production. In contrast, our approach focuses on the opposite chain of causality. The heterogeneity in our model occurs on the preference side rather than on the supply side. This allows us to study the *effect of* an unequal distribution of income on the incentives to innovate.

Our paper is clearly also related to the recent literature on inequality and growth. Much of this literature has either focused on the role of capital market imperfections, (see Galor and Zeira (1993), Banerjee and Newman (1993), Aghion and Bolton (1997), Galor and Tsiddon (1997), Galor and Moav (2003), and others) or on political mechanisms (Bertola (1993), Persson and Tabellini (1994), Alesina and Rodrik (1994), and others) or on a combination of these (Bénabou, 1996, 2000, 2004). In contrast, the present paper focuses on the role of inequality for the dynamics of an innovator's demand and does neither rely on imperfect capital markets nor on politico-economic arguments. There is a small literature that emphasizes the role of demand for the incentive to innovate (e.g. Falkinger, 1990, 1994, Chou and Talmain, 1994, Li, 1996, Zweimüller, 2000, Glass, 2001, and Zweimüller and Brunner, 2004). However, none of these papers focuses on the role of inequality on the structure of prices and the exclusion of poor consumers from the innovator's markets – issues which are central to our model of income distribution and demand-induced inventions.

The importance of a hierarchic structure of demand for the adoption of new technologies is emphasized also by many dual economy models. Murphy, Shleifer, and Vishny (1989) provide a static model of technology adoption that specifies preferences in a similar form as we do. However, in their model, prices are exogenously given and their focus is exclusively on the market size effect.

The empirical literature on demand-induced technical change is even more scarce. Most research has focused on the pharmaceutical industry. Kremer (2001a, 2001b) has a number of papers why research on vaccines for Malaria, tuberculosis, and the strains of HIV is so minimal - despite the fact that very many individuals in the Third World suffer from these diseases. His main explanation relies on the demand side: Potential vaccine developers fear that they would not be able to sell enough vaccine at a sufficient price to recoup their research expenses.[1] Acemoglu and Linn (2003) investigate the effect of potential market size on innovation of new drugs and find substantial effects of potential market size.

[1] An additional explanation for this result lies in the difficulty to enforce property rights on medicaments in developing countries.

3 The Demand Side

3.1 Preferences

Consider an economy with many potentially producible differentiated products indexed by a continuous index $j \in [0, \infty)$. Consumers' preferences over these differentiated goods are 'hierarchic' in the sense that are ranked according to their priority in consumption. Goods with a low index have high priority, whereas goods with a higher index have lower priority. We assume separability and model the hierarchy as follows. There is a 'baseline' utility function, $v(c)$, the same for all differentiated goods, and a 'weighting function $\xi(j)$ with $\xi'(j) < 0$. This yields a ranking of the various products: low-j goods get a high weight (have high priority in consumption) whereas higher-j goods have lower priority.

We proceed by restricting the functions $v(\cdot)$ and $\xi(\cdot)$. We assume that the consumer's choice of a differentiated product is a 'take-it or leave-it' decision: either a product is consumed in quantity 1 or is not consumed. This allows us to normalize the baseline utility to $v(0) = 0$ and $v(1) = 1$. Furthermore, we assume that the hierarchy function takes the form $\xi(j) = j^{-\gamma}$ with $\gamma \in (0, 1]$. The first restriction is primarily for tractability and analytical convenience. The second restrictions is necessary to ensure that the model exhibits a balanced growth path.

The preferences over the differentiated products can thus be represented by $\tilde{u}(\{c(j)\}) = \int_0^\infty j^{-\gamma} c(j) dj$ where $c(j) \in \{0, 1\}$ indicates whether good j is consumed or not. When a consumer purchases the first N goods in the hierarchy the utility is given by $\int_0^\infty j^{-\gamma} c(j) di = \int_0^N j^{-\gamma} di = \frac{N^{1-\gamma}}{1-\gamma}$. (The restriction $\gamma \in [0, 1)$ makes sure that the integral $\int_0^N j^{-\gamma} di$ does not diverge). Would all goods have the same price, the highest utility arises from consuming all goods in the interval $[0, N]$, it is also evident that the utility integral is finite for any arbitrary bundle of goods with measure N: any arbitrary interval of measure N (or sub-intervals that sum up to measure N) yields instantaneous utility larger than 0 but lower than $\frac{N^{1-\gamma}}{1-\gamma}$.

Apart from the sector of differentiated products, there exists also a second sector that produces a homogeneous good x that can be consumed in continuous amounts. (A possible different interpretation of x is leisure). Total utility depends on the quantity of homogenous goods and the set of differentiated products. Assuming that the two types of goods are linked by a Cobb-Douglas relationship with parameter ν, where $0 \leq \nu < 1$,[2] and that the first N

[2]The Cobb-Douglas implies that expenditure shares are *per se* not systematically related to the income level.

goods of the consumption hierarchy are available on the market,[3] we can express the total utility as $u(x, \{c(j)\}) = x^\nu \int_0^N j^{-\gamma} c(j) dj$.

Consumers have an infinite time horizon. Their objective function can be written as

$$U(t) = \int_t^\infty \left[x(s)^\nu \int_0^{N(s)} j^{-\gamma} c(j, s) dj \right] e^{-\rho(s-t)} ds. \quad (1)$$

where ρ denotes the rate of time preference.

3.2 Endowments

All consumers have the same objective function (1) but differ in their endowments. We assume there are two types of consumers, poor P and rich R, with population size β and $1-\beta$, respectively. All households derive income from working and from shares in profits that accrue in the monopolistic firms. We assume further that each household has the same income composition (identical labor and profit shares). Hence the ratio of the income level of the poor relative to per capita income is $\theta_P < 1$, and the corresponding ratio of the rich is $\theta_R > 1$. Obviously the income shares of poor and rich must sum up to unity, so we have $(1-\beta)\theta_R + \beta\theta_P = 1$. Taking $\vartheta \equiv \theta_P$ as the exogenous parameter, we have $\theta_R = (1-\beta\vartheta)/(1-\beta)$. Hence the two parameters β and ϑ fully characterize the income distribution.[4]

We note that the assumption of identical income shares is restrictive and can be disputed both on empirical and theoretical grounds. It implies, for instance, that the distribution of income and the distribution of wealth are identical, whereas in reality the latter is more unequal than the former. Furthermore it implies that there is no feedback from the functional distribution to the personal distribution of income. There are basically two reasons why we adopt this

We will see below, however, that in equilibrium, rich and poor consumers have a different expenditure share of the homogenous product because the poor consume less differentiated products and face a different average price level for these products than the rich.

[3]We are making a shortcut here. More generally, we could assume that a bundle of goods with measure $N(t)$ is available at date t but this measure does not need to coincide with the interval $[0, N]$ in the hierarchy. In other words the inherited set of goods may be such that there are 'holes' in the hierarchy. We neglect this possibility here because we are interested in the analysis of a balanced growth equilibrium. In equilibrium the introduction of new goods will follow the hierarchy, as the most recent innovator always introduces the good with highest priority among the goods that have not yet been invented.

[4]The corresponding Lorenz-curve is piecewise linear with slope ϑ up to population share β; and slope $(1-\beta\vartheta)/(1-\beta)$ for population shares between β and 1.

assumption. First, this assumption allows us to keep that analysis tractable. In particular, this assumption implies that θ_P and θ_R do not change over time but remain truly exogenous. Second, this assumption nevertheless allows us to highlight the interesting mechanisms how inequality affects the growth. While the analysis would become more complicated, the gain in economic insight would be minor. A final reason is that, under our specification of preferences, an identical income composition implies that all households have the same savings rate. In other words, any impact of income inequality in our model arises due to hierarchic preferences. None of the inequality effects is due to differences in the propensities to save between rich and poor consumers.

3.3 Consumption Choices

Under our assumptions the intertemporal budget constraint of household i can be written as follows

$$\int_t^\infty \left[p_x(s)x_i(s) + \int_0^{N(s)} p(j,s)c_i(j,s)dj \right] e^{-R(s,t)}ds \leq \int_t^\infty w(s)l_i e^{-R(s,t)}ds + V_i(t), \tag{2}$$

where $p_x(s)$, $N(s)$, $p(j,t)$, and $w(s)$ denote, respectively, the price of the homogenous good, the mass of available differentiated products, the price of variety j, and the wage rate at date s. $R(s,t) = \int_t^s r(\tau)d\tau$ is the cumulative discount factor between dates t and s, l_i is the (time-invariant) labor endowment of household i, and $V_i(t)$ is the initial wealth level owned by the household $i \in \{P, R\}$.

The household maximizes (1) subject to the budget constraint (2). Setting up the Lagrangian, it is straightforward to obtain the first order conditions, respectively for $c_i(j,s)$ and $x_i(s)$

$$\begin{aligned} c_i(j,s) &= 1 \qquad p(j,s) \leq x_i(s)^\nu j^{-\gamma} \frac{e^{R(s,t)-\rho(s-t)}}{\mu_i} \equiv q_i(j,s), \\ c_i(j,s) &= 0 \qquad p(j,s) > q_i(j,s), \\ v x_i(s)^{\nu-1} \int_0^{N(s)} j^{-\gamma} c_i(j,s)dj &= \frac{\mu_i}{e^{R(s,t)-\rho(s-t)}} p_x(s). \end{aligned} \tag{3}$$

where μ_i is the Lagrangian multiplier, the marginal utility of wealth at the initial date t. (This can be translated into the more familiar 'time-s' marginal utility of wealth $\lambda_i(j,s) = \mu_i e^{-R(s,t)+\rho(s-t)}$). The first two equations in (3) state that, at date s, consumer i will purchase the differentiated good j, if its price $p(j,s)$ does not exceed the willingness to pay - which we denote by $q_i(j,s)$; and will not purchase otherwise. The third equation in (3) determines the optimal amount of consumption of the homogenous good x at date s.

Let us take a closer look at $q_i(j,s)$, consumer i's willingness to pay. Obviously, $q_i(j,s)$ is larger the lower the position of good j in the hierarchy (= the higher the priority of good j). Furthermore, $q_i(j,s))$ is the higher the smaller is the consumer's marginal utility of wealth $\lambda_i(j,s) = \mu_i e^{-R(s,t)+\rho(s-t)}$. Obviously, rich consumers have a lower marginal utility of wealth and their willingness to pay is higher. Finally, the willingness to pay is also increasing in the consumption level of the homogenous goods as differentiated and homogenous goods are gross complements.

4 Technology and price setting

4.1 Production technology and technical progress

The supply side of the model is simple. Labor is the only production factor and the labor market is competitive. The market clearing wage at date t is denoted by $\tilde{w}(t)$. The differentiated products are produced by monopolistic firms under increasing returns to scale. Before a good can be produced the firm has to make an 'innovation'. This gives the firm exclusive access to the blueprint of the new good and guarantees monopoly position.[5] The innovation cost are modelled by a set-up cost equal to $\tilde{F}(t)$ labor units. Once this set-up cost has been incurred, the firm has access to a linear technology that require $\tilde{b}(t)$ units of labor to produce one unit of output.

Innovations imply technical progress. We assume that the knowledge stock of this economy equals the number of known designs $N(t)$. The labor coefficients in the sector that produces differentiated goods are inversely related to the stock of knowledge. Hence we have $\tilde{F}(t) = F/N(t)$ and $\tilde{b}(t) = b/N(t)$ where $F > 0$ and $b > 0$ are exogenous parameters. Wages grow pari passu with productivity, $\tilde{w}(t) = wN(t)$, where $w > 0$ is a constant. Hence the cost of production of differentiated products stay constant over time as $\tilde{w}(t)\tilde{F}(t) = wF$ and $\tilde{w}(t)\tilde{b}(t) = wb$ are constant over time.[6]

Finally, the homogenous sector produces output with a linear technology that requires b_x

[5] By assumption, we rule out that there is no duplication. So when a new good is 'invented' there is one and only one firm that incurs that fixed cost and captures the respective market.

[6] We choose the marginal production cost in the differentiated sector as the numeraire $wb = 1$, then $w = 1/b$.

labor units to produce one unit of output. There is no technical progress in this sector. The labor coefficient b_x is constant over time. The output market is competitive, prices equal marginal cost of production $p_x(t) = \tilde{w}(t)b_x = wb_xN(t)$.

4.2 Prices of the differentiated goods

Producers of differentiated products are in a monopoly position and can set prices above marginal cost of production. We take the marginal production cost as the numeraire so $wb = 1$. In order to determine the monopoly price we need the monopolist's demand function. Consider the demand for good j. Consumption is a binary choice and the level of demand at price $p(j,t)$ depends on how many consumers are willing to purchase good j at that price. Obviously the market demand function of a monopolistic producer is a step function (Figure 1). At prices that exceed the willingness to pay for the rich, $p(j,t) > q_R(j,t)$, demand is zero and the demand curve in Figure 1 coincides with the vertical axes. For prices that do not exceed the willingness to pay for rich, but are strictly larger than the willingness to pay for the poor, $p(j,t) \in (q_P(j), q_R(j)]$, market demand equals the population size of the rich $1-\beta$. Finally, for prices lower than or equal to the willingness to pay of the poor $p(j,t) \leq q_P(j,t)$ market demand equal the size of the whole population in the economy which we have normalized to unity.[7]

It is obvious that monopolist will set charge the willingness to pay of the rich and sell only to the rich (point A in Figure 1) or charge the willingness to pay of the poor and sell to the whole population (point B in Figure 1), whichever yields the higher profits. The corresponding profit levels are, respectively, $[q_R(j,t) - 1](1-\beta) \equiv \Pi_R(j,t)$ (point A) and $[q_P(j,t) - 1] \equiv \Pi_{tot}(j,t)$ (point B).

Figure 1.

Suppose the first $N(t)$ products of the consumption hierarchy are supplied at date t. Which firms will choose high prices and sell to the rich? Which firms will choose low prices and serve the whole population? To answer this question we note first that, a situation where all firms

[7] Obviously, if there are more types of consumers, there are more such kinks, and in the case of continuous distribution we have a smooth demand function. In any case, under the take-it or leave-it assumption the shape of the demand function reflects the distribution of the consumers' willingnesses to pay.

charge high prices and only the rich buy goods from monopolistic producers, cannot be an equilibrium. If the poor would not buy any differentiated products at all, our specification of preferences implies that their willingness to pay would become infinitely large for goods $j \to 0$. Hence there must be some j such that $\Pi_{tot}(j) \geq \Pi_R(j)$ or equivalently, $q_P(j) - q_R(j)(1-\beta) \geq \beta$, which implies that $q_P(j)/q_R(j) > 1-\beta$. This leads us to the following

Lemma 1 ***(Consumption follows the hierarchy)*** *Prices are set such that for all goods $j \in [0, N_P(t)]$, $p(j,t) = q_P(j,t)$, and for all $j \in (N_{P,}(t), N_R(t)]$ we have $p(j,t) = q_R(j,t)$, where $0 < N_P(t) \leq N_R(t) \leq N(t)$.*

Proof. (For convenience, we omit time-indices). We know from equation (3) that $q_i(j) = x_i^{\nu} j^{-\gamma}/\lambda_i$. (Recall the definition of $\lambda_i(j,s) = \mu_i e^{-R(s,t)+\rho(s-t)}$). Hence it is straightforward to calculate $\partial \Pi_R(j)/\partial j = -(\gamma/j)\, q_R(j)(1-\beta)$ and $\partial \Pi_{tot}(j)/\partial j = -(\gamma/j)\, q_P(j)$. Since $q_P(j)/q_R(j) > 1-\beta$, we see that $\partial \Pi_R(j)/\partial j > \partial \Pi_{tot}(j)/\partial j$, which means that the difference $\Pi_{tot}(j) - \Pi_R(j)$ decreases as we move along the consumption hierarchy. Since the poor always consume a positive subset of the differentiated goods, there exist a good $N_P > 0$ such that for any $j \leq N_P$ we have $\Pi_{tot}(j) \geq \Pi_R(j)$. ■

Hence Lemma 1 implies that the prices of the differentiated products are

$$p(j,t) = \begin{cases} q_P(j,t) & j \in [0, N_P(t)] \\ q_R(j,t) & j \in (N_P(t), N_R(t)]. \end{cases} \tag{4}$$

The poor consume all goods $j \in [0, N_P]$ and the rich consume all goods $j \in [0, N_R]$ where $0 < N_P \leq N_R \leq N$. This means 'consumption follows the hierarchy' in the sense that consumer i purchases only the first N_i products in the hierarchy and no products $j > N_i$. This is an intuitive result: the goods with lower priority (high-j goods) are priced such that only the rich can afford them.

Proposition 1 *The equilibrium is characterized by three regimes.*

(i) $0 < N_P < N_R = N$: The rich consume all available products and there is 'market exclusion' of the poor. If the willingnesses to pay of the rich and the poor are sufficiently different, we get $0 < N_P < N_R$. There is 'market exclusion' of the poor: The poor are excluded from participation in certain markets even though they have a willingness to pay that exceeds the marginal costs of production. As we will show below, this first scenario will prevail if the willingnesses to pay are sufficiently different.

(ii) $0 < N_P = N_R = N$: All agents consume all products. In that regime, no monopolist finds it optimal to sell to the rich only. This will be optimal if consumers are very similar. Then, x_P/x_R and (μ_R/μ_P) are arbitrarily close to unity. This implies that the relative willingness to pay $q_P(j)/q_R(j) = (x_P/x_R)^\nu (\mu_R/\mu_P)$ is close to 1.

(iii) $0 < N_P < N_R < N$: Neither the rich nor the poor can afford all $N(t)$ goods and there is market exclusion of the poor. This is the case if $q_R(N) < 1$. This third scenario is possible if the research costs to invent a new good are very low.

(iv) The regime $0 < N_P = N_R < N$ cannot be an equilibrium. In such a regime $[q_R(N_P) - 1](1 - \beta) = q_P(N_P) - 1$ would hold. As $q_R(j)/q_P(j) > 1$ we must have $q_R(N_R) > q_P(N_R) > 1$. Hence, the willingness to pay for good $N_R + \varepsilon$, where ε is arbitrarily close to zero, would also exceed one due to the continuity of $q_R(j)$. Consequently, there would be positive profits from selling good $N_R + \varepsilon$ and the claim follows.

Corollary 1 *If $\nu = 0$, the poor will always be excluded from some goods in equilibrium.*

Proof. If $N_P = N_R = N$ rich and poor would consume exactly the same. With $\nu = 0$ there are no expenditure for x-goods. The rich would have income left causing their marginal willingness to pay be infinitely large. ■

The more general message of the above Lemma 1 is that the structure of prices is determined by the distribution of wealth among the households. This is a result that is absent from the standard monopolistic competition due to the assumption of homothetic preferences: total market demand there is independent of the income distribution and has therefore no effect of the structure of prices. And most previous attempts to combine a necessity ranking of the products with market power have imposed the assumption of uniform mark-ups for all products (see Murphy, Shleifer, Vishny (1989), and Zweimüller (2000)).

Finally, we observe further that the distribution of wealth affects not only the choice of prices and the profits of individual firms but it also affects aggregate profits. In other words, the *personal* distribution of income (exogenous in this model) endogenously determines the *functional* distribution.

5 Balanced growth: the regime $N_P < N_R = N$

5.1 The allocation of resources across sectors

The economy's resources consist of the stock of knowledge $N(t)$ and homogeneous labor supplied by each household in the economy which we have normalized to 1. At any date t, $N(t)$ is predetermined but affects current productivities $\tilde{b}(t)$ and $\tilde{F}(t)$. The allocation of labor resource across production sectors is endogenously determined. We denote by L_N the number of workers employed in the sector producing the differentiated hierarchical products and by L_x the number of workers employed in the sector producing the homogenous good. Moreover, as we study a model of (costly) innovation and growth, at any date t, resources are also employed in an R&D sector that develops blueprints for new products. We denote by L_R the number of workers that are employed to conduct such innovative activities.

First, let us consider employment in the production of the differentiated products. The labor demand in this sector depends is given by $L_N(t) = \int_0^{N(t)} [b/N(t)] \left[\beta c_P(j,t) + (1-\beta) c_R(j,t)\right] dj$. We are concerned with the regime where the poor consume the first $N_P(t) < N(t)$ products in the consumption hierarchy, whereas the rich consume all varieties currently available $N_R(t) = N(t)$. In that case we get $L_N(t) = b\left[\beta n_P(t) + (1-\beta)\right]$ where $n_P(t) = N_P(t)/N(t)$ denotes the fraction of available differentiated products consumed by the poor. Second, employment in the production of the homogenous good is given by $L_x(t) = b_x \left[\beta x_P(t) + (1-\beta) x_R(t)\right]$. And third, employment in the research sector depends on $\dot{N}(t)$, the level of innovation activities at date t. As introducing a new product requires $F/N(t)$ labor units, we get $L_R = F\dot{N}(t)/N(t) = Fg(t)$.

A perfect labor market ensures that the labor supply is fully employed at each date, so $1 = L_N + L_x + L_R$. Using the above expressions the economy's resource constraint can be written as

$$1 = b\left[\beta n_P(t) + (1-\beta)\right] + b_x \left[\beta x_P(t) + (1-\beta) x_R(t)\right] + Fg(t). \tag{5}$$

The dynamic analysis below focuses on a balanced growth path, along which the allocation of labor across the three sectors stays constant over time. >From equation (5) it is obvious, that a balanced growth path is only possible if $n_P(t) = n_P$, $x_P(t) = x_P$, $x_R(t) = x_R$, and $g(t) = g$ do not change over time.

5.2 The interest rate

We can make use of these balanced growth properties to determine the interest rate along this path. To calculate the interest rate we use equation (3) and Lemma 1. This Lemma says that consumer i purchases the menu $[0, N_i(t)]$ of the consumption hierarchy. The third line of equation (3) can be rewritten as $\nu x_i(s)^{\nu-1}\frac{N_i(s)^{1-\gamma}}{1-\gamma} = \mu_i e^{-R(s,t)+\rho(s-t)} p_x(s)$. Taking logs and the derivative with respect to time s yields (note that μ_i does not depend on s)

$$(\nu - 1)\frac{\dot{x}_i(s)}{x_i(s)} + (1-\gamma)\frac{\dot{N}_i(s)}{N_i(s)} = -r(s) + \rho + \frac{\dot{p}_x(s)}{p_x(s)}.$$

Along a balanced growth path, the menu of differentiated goods increases the at the same rate as $N(t)$ for both types of consumers. This rate is constant and given by g. Moreover, we know that the amounts of the homogenous good $x_i(s)$ do not change over time and that the prices for these products $p_x(s) = wb_x N(t)$ increase at rate g. We can use these observations and solve for the interest rate

$$r(s) = \rho + g\gamma, \tag{6}$$

which, unsurprisingly, is also constant along the balanced growth path. Obviously, equation (6) is the equivalent in our model to the familiar Euler equation in the standard growth model. Just like the elasticity of marginal utility in the standard model, the hierarchy parameter γ tells us how an increase in the range of consumed goods affects the utility flow.

5.3 Changes in prices along the balanced growth path

We have already mentioned that, along the balanced growth path, the price for the *homogenous* good increases at rate g. How do the prices for a *differentiated* product evolve along this path? To get an answer to this question we use the first two lines in equation (3). We know from Lemma 1 that the price of goods $j \in [0, N_P(t)]$ equals the willingness to pay of the poor $q_P(j,s) = x_P(s)^\nu j^{-\gamma} e^{R(s,t)-\rho(s-t)}/\mu_P$ and that the price for goods $j \in (N_P(t), N_R(t)]$ equals the willingness to pay of the rich $q_R(j,s) = x_R(s)^\nu j^{-\gamma} e^{R(s,t)-\rho(s-t)}/\mu_R$. Suppose good j becomes affordable to the poor at date s, $j = N_P(s)$, then the price of good j makes a discrete jump, from $q_R(N_P(s), s)$ to $q_P(N(s), s)$. Now consider the change in prices before and after that date. Taking logs of the above expressions for $q_P(j,s)$ and $q_R(j,s)$ and the derivative with respect to time s, we see that

$$\frac{\dot{p}(j,s)}{p(j,s)} = r - \rho = g\gamma,$$

where the first string follows from the balanced growth properties $\dot{x}_i/x_i = 0$ and $\dot{N}_i/N_i = g$, and the second string follows from equation (6). We can make a slightly different thought experiment and the evolution of the price of good $N_i(s)$, the price of the good with least priority purchased by consumer i at date s. Setting $j = N_i(s)$ in the above expressions for $q_P(j,s)$ and $q_R(j,s)$, taking logs and the derivative with respect to time s yields

$$\frac{\dot{p}(N_i(s),s)}{p(N_i(s),s)} = -\gamma g + r - \rho = 0.$$

where, again, the second string of the above equation uses the balanced growth properties $\dot{x}_i/x_i = 0$ and $\dot{N}_i/N_i = g$, and the third string follows from equation (6).

The different evolutions of $p(j,s)$ and $p(N_i(s),s)$ highlight the implications of our formulation of a consumption hierarchy. Prices are depend on the willingness to pay for the various products, and the willingness to pay depends on the position of product j *relative to available range of products,* $j/N(s)$. Obviously, the relative position of a particular good j decreases over time, as $N(s)$ increases. (The decrease in the relative position in the consumption hierarchy reflects the idea become relatively more necessary.) Holding the relative position in the hierarchy constant (as we do in the case when we look at the price of good $N_i(s)$ at date s (this relative position is $n_P < 1$ and $n_R = 1$ in the regime $N_P < N_R = N$) we see that the price remains constant. In sum, take a good that is introduced at date s. This good starts out with price $p(N(s),s)$, increases at rate $g\gamma$ until the date when the poor have become sufficiently rich. At that date the price makes a discrete jump down to the willingness to pay of the poor and increases at rate $g\gamma$ thereafter.

Two comments on the evolution of prices are in order. First, it is obvious that the discontinuous evolution of prices with a discrete jump when the poor start to purchase, is due to our assumption of two groups of consumers. It is obvious, that our analysis can be extended to the case of many groups. In that case there are many such changes in prices in order to attract additional groups or customers. Furthermore, we have not allowed for learning-effects in the production of a particular variety. To the contrary, the evolution of prices reflects the fact that a particular good experiences a higher willingness to pay because the consumers have already satisfied. The *relative* position of good j in the hierarchy, $j/N(t)$, decreases. In this sense, a good that was previously a luxury good, has become a necessity. This is reflected in increasing willingnesses to pay and rising prices for this product.

5.4 The innovation process

Up to now we have taken a continuous introduction of new products (and corresponding increases in productivity) for granted. However, our aim is to study the effects of income inequality on innovation and growth of the income distribution that are transmitted from demand side. Hence we have to look at the impact of income inequality on the incentives for a potential innovator to introduce a new product.

It is assumed that there is free entry into the R&D sector and the equilibrium is a situation of zero profits. That is, the value of an innovation equals to the cost of an innovation. The cost of an innovation is given by wF. It remains to determine the value of an innovation. Clearly, innovation efforts will be targeted towards those goods for which consumers are willing to pay most. Hence the most recent innovator always introduces the good with the highest priority in the set of goods that are not yet available. In this sense, the R&D process follows the consumption hierarchy.

The value of an innovation equals the discount value of the profit flow following the successful introduction of a new product. In our model, the profit of a particular innovator is increasing over time, because both demand and prices increase over time. Consider first the evolution of demand. A firm that incurs the set-up costs and is granted a patent of infinite length[8] at date t has initial demand $1-\beta$ (our focus is the case $N_P < N_R = N$). Growth in incomes lead to an increase in the willingness to pay of the poor. Hence after some time the innovator will find it optimal to charge the willingness to pay of the poor and sell to all consumers. Denote by Δ the time interval during which only the rich purchase a new product. Obviously, Δ must satisfy $N_P(t+\Delta) = N(t)$. Along a balanced growth path, where N_P grows at the constant rate g and we can write $N_P(t)e^{g\Delta} = N(t)$. Taking logs and solving for Δ yields

$$\Delta = -\ln[N_P(t)/N(t)]/g = -\ln n_P/g,$$

[8]That an innovator stays on the market forever is a simplifying assumption. We could introduce, for instance, finite patent protection and assume that the market become competitive once the patent has expired. Our main conclusions would remain unchanged, as long as patents expire before the poor can afford the good. If patents expire earlier, it is only the willingsness to pay of the rich that counts for the incentive to innovate.

Note that $\Delta > 0$ as $n_P < 1$. Obviously, the duration Δ during which an innovator sells only to the rich is long if (i) the poor are very poor (so the fraction of goods the poor can afford n_P is small) and (ii) the growth rate g is low.

It turns out convenient to express the price of good N_P in terms of the price of the most recent innovator. (For simplicity, we omit time indices in the present paragraph). We know from equation (3) that $q_R(N_P) = n_P^{-\gamma} q_R(N)$. Furthermore good N_P is the good of the monopolist who earn the same profit when selling to the whole population at the willingness to pay of the poor as when selling to rich at a high price. Hence N_P satisfies $q_P(N_P) - 1 = [q_R(N_P) - 1](1-\beta)$. Using $q_R(N) = p$ and $q_R(N_P) = n_P^{-\gamma} q_R(N)$ we can solve the arbitrage condition for the price of good N_P

$$q_P(N_P) = \beta + (1-\beta) n_P^{-\gamma} p.$$

Note that, along the balanced growth path, $q_P(N_P(t), t)$ is constant over time as both n_P and p do not change over time.

The equilibrium in the R&D sector requires that the costs of an innovation wF equal the value of an innovation, given by the present value of the profit flow. From our above analysis, this profit flow equals $(1-\beta)\left(pe^{g\gamma(s-t)} - 1\right)$ at dates $s \in [t, t+\Delta)$ (when the innovator sells on the rich) and equals $\left[\beta + (1-\beta) n_P^{-\gamma} p\right] e^{g\gamma(s-t-\Delta)} - 1$ at dates $s \geq t + \Delta$ (when the innovator sells to the whole population). The equilibrium condition of the R&D sector can then be written as

$$wF = (1-\beta)\left(\frac{p}{\rho} - \frac{1}{r}\right) + \beta n_P^{\gamma}\left(\frac{n_P^{\rho/g}}{\rho} - \frac{n_P^{r/g}}{r}\right), \tag{7}$$

where we have used $r = \rho + g\gamma$ from equation (6).

5.5 Solving for the equilibrium growth rate

We continue to focus on the regime where the rich but not the poor purchase the differentiated products available on the market, $N_P < N_R = N$. To solve for the balanced growth equilibrium, we use Lemma 1 to rewrite the first order condition (3) for the consumption choices of the rich and the poor as

$$x_P = \frac{\nu N}{1-\gamma} \frac{\left[\beta + (1-\beta) p n_P^{-\gamma}\right]}{w b_x} n_P, \text{ and} \tag{8}$$

$$x_R = \frac{\nu N}{1-\gamma} \frac{p}{w b_x}. \tag{9}$$

Furthermore, we use Lemma 1, equation (8) and (9), and equation (6) to rewrite the budget constraints of rich and the poor consumers, respectively, as

$$\begin{aligned} wl_P + (\rho - (1-\gamma)\,g)\frac{V_{tP}}{N_t} &= \left[\beta n_P + (1-\beta)pn_P^{1-\gamma}\right]\frac{1+\nu}{1-\gamma}, \text{ and} \\ wl_R + (\rho - (1-\gamma)\,g)\frac{V_{tR}}{N_t} &= \left[\beta n_P + (1-\beta)pn_P^{1-\gamma}\right]\frac{1}{1-\gamma} + \overline{p}\frac{1+\nu-n_P^{1-\gamma}}{1-\gamma}. \end{aligned}$$

Dividing the former equation by the latter and making use of our assumption on the endowment distribution $l_P/l_R = V_{tP}/V_{tR} = \theta_P/\theta_R = \vartheta(1-\beta)/(1-\vartheta\beta)$ yields an equation that can be solved for p

$$p = \frac{\left[\nu\beta + (1+\nu)\,\frac{(1-\vartheta)\beta}{(1-\beta)\vartheta}\right] n_P}{1+\nu - \left[\nu(1-\beta) + \frac{1-\vartheta}{\vartheta}\,(1+\nu)\right] n_P^{1-\gamma}} \equiv \varphi(n_P), \text{ with } \varphi'(n_P) > 0. \tag{10}$$

We directly see that p is a monotonically increasing in n_P. The intuition is straightforward: *For a given degree of inequality* (as represented by the exogenous parameters β and ϑ), a situation where the poor want to purchase a larger range of the differentiated products (n_P is higher) goes hand in hand with a situation where the rich are willing to pay a higher price for the product of the most recent innovator (p is higher). Note also that in regime $N_P < N_R = N$ we have $n_P < 1$ and $p > 1$. We see from equation (10) how this restricts the relevant range of p and n_P. We immediately see from equation (10) that $\varphi(1) > 1$. Moreover, we see that there exists a critical value of n_P, call it m, such that $\varphi(m) = 1$. Hence the relevant range for the regime $N_P < N_R = N$ is $p \in [1, \varphi(1)]$ and $n_P \in [m, 1]$.

To determine the growth rate we use equation (5), replace x_P and x_R in by the equations (8) and (9), and p by $\varphi(n_P)$ from (10). This gives us a first equation in the two endogenous variables n_P and g

$$1 = gF + b\,(\beta n_P + 1 - \beta) + \frac{\nu}{1-\gamma} b \left(\beta^2 n_P + \beta(1-\beta)n_P^{1-\gamma}\varphi(n_P) + (1-\beta)\varphi(n_P)\right). \tag{11}$$

Next we use equations (6) and (10) and rewrite the R&D equilibrium condition (7) to get the second equation in n_P and g

$$wF = (1-\beta)\left(\frac{\varphi(n_P)}{\rho} - \frac{1}{\rho+\gamma g}\right) + \beta n_P^{\gamma}\left(\frac{n_P^{\rho/g}}{\rho} - \frac{n_P^{(\rho+\gamma g)/g}}{\rho+\gamma g}\right). \tag{12}$$

6 The other regimes

6.1 Regime $N_P < N_R < N$

When neither the poor nor the rich can afford all differentiated products available on the market we are in regime $N_P < N_R < N$. The equilibrium conditions differ from the above regime in two respects. First, in this scenario goods $j \in (N_R, N]$ have no demand. However, the structure of prices can be expressed in a similar way as before by relating all other prices of the differentiated products to the price of good N_R, the good with least priority that is purchased by the rich. Obviously the price of good N_R must equal marginal cost, that is $p(N_R) = 1$. If $p(N_R) > 1$ it would be profitable for a firm $j > N_R$ to start production since the willingness to pay of the rich would be above the production cost. Noting that all $j \in (N_P(t), N_R(t)]$ are priced at the willingness to pay of the rich and all $j \in [0, N_P(t)]$ are priced at the willingness to pay of the poor. Using (3), $p(N_R) = 1$, and (from monopolist N_P's arbitrage condition) $p(N_P) = (1-\beta)(N_P/N_R^{-\gamma}) + \beta = (j/N_R^{-\gamma})$ the prices for all goods can be calculated.

To describe the general equilibrium we proceed in a similar way as before. We use Lemma 1 to rewrite the first order condition (3) for the consumption choices of the rich and the poor as

$$x_P = \frac{\nu N}{1-\gamma} \frac{\beta + (1-\beta)\left(\frac{n_P}{n_R}\right)^{-\gamma}}{wb_x} \frac{n_P}{n_R}, \text{ and} \tag{13}$$

$$x_R = \frac{\nu N}{1-\gamma} \frac{1}{wb_x} n_R. \tag{14}$$

Furthermore, we use Lemma 1, equations (13) and (14) and equation (6) to rewrite budget constraints as

$$wl_P + (\rho - (1-\gamma)\,g)\frac{V_{tP}}{N_t} = \left[\beta \frac{n_P}{n_R} + (1-\beta)\left(\frac{n_P}{n_R}\right)^{1-\gamma}\right] \frac{1+\nu}{1-\gamma}, \text{ and}$$

$$wl_R + (\rho - (1-\gamma)\,g)\frac{V_{tR}}{N_t} = \left[\beta \frac{n_P}{n_R} + (1-\beta)\left(\frac{n_P}{n_R}\right)^{1-\gamma}\right] \frac{1}{1-\gamma} + \frac{1+\nu-\left(\frac{n_P}{n_R}\right)^{1-\gamma}}{1-\gamma}.$$

Dividing the former equation by the latter, defining $m = n_P/n_R$, and using our distributional assumption yields

$$\frac{1-\vartheta}{(1-\beta)\,\vartheta} = \frac{\left(1+\nu-m^{1-\gamma}\right) - \nu\left(\beta m + (1-\beta)m^{1-\gamma}\right)}{\left(\beta m + (1-\beta)m^{1-\gamma}\right)(1+\nu)}. \tag{15}$$

The right hand side of equation (15) is monotonically declining in m, starting from ∞ as $m = 0$ and reaching 0 as $m = 1$. Hence, there is a unique value of $m \in (0, 1)$ that satisfies equation (15). This equation says that, for a given degree of inequality in the endowment distribution, there is a constant ratio n_P/n_R that prevails in regime $N_P < N_R < N$. This ratio is independent of the equilibrium growth rate. Furthermore it depends only on distribution and preference parameters, but not on technological parameters.

In regime $N_P < N_R < N$ the resource constraint is only slightly different from the one in regime $N_P < N_R = N$ (see equation (5)) and given by $1 = b\,[\beta n_P + (1-\beta)n_R] + b_x\,[x_P + (1-\beta)x_R] + gF$. We can use Lemma 1, replace x_P and x_R by equations (13) and (14) and using the fact that $n_R = n_P/m$ to get a first equation in g and n_P

$$1 = gF + b\frac{n_P}{m}\,(\beta m + 1 - \beta) + \frac{\nu b}{1-\gamma}\frac{n_P}{m}\left[\beta^2 m + \beta(1-\beta)m^{1-\gamma} + 1 - \beta\right]. \qquad (16)$$

To get the second equation we have to focus on the R&D equilibrium condition in the regime $N_P < N_R < N$.

Obviously, in this regime the most recent innovator has initially no demand and has to wait until the rich have a willingness to pay is at least as large to cover the production cost. (The firm has an incentive to undertake the necessary R&D efforts as the innovation grants a patent, and prevents other innovators from entering the market.) How long is the waiting time? On a balanced growth path with rate g, this waiting time δ is defined by the equation $N_R(t)e^{g\delta} = N(t)$, or equivalently, $\delta = -\ln n_R/g$. Obviously, the waiting time δ is short when growth is high and/or when the rich can afford a high fraction of the available products. The life-cycle of the innovator at date t looks as follows. Demand is zero during the time interval $[t, t+\delta)$, $1-\beta$ during the interval $[t+\delta, t+\Delta)$, and 1 from $t+\Delta$ onwards. The price is 1 at date $t+\delta$, increases at rate $g\gamma$ until date $t+\Delta$, when it falls to $\beta + (1-\beta)m^{-\gamma}$, and increases at rate $g\gamma$ thereafter. The production cost remain constant over time. It is straightforward to calculate the present value of this profit flow. Using equation (6), $\delta = -\ln n_R/g$, $\Delta = -\ln n_R/g$, and $n_R = n_P/m$ the R&D equilibrium condition in regime $N_P < N_R < N$ can be expressed in terms of g and n_P

$$wF = \left((1-\beta)\left(\frac{1}{\rho} - \frac{1}{\rho+\gamma g}\right) + \beta m^{\gamma}\left(\frac{m^{\rho/g}}{\rho} - \frac{m^{(\rho+\gamma g)/g}}{\rho+\gamma g}\right)\right)\cdot\left(\frac{n_P}{m}\right)^{\frac{\rho+g\gamma}{g}} \qquad (17)$$

6.2 The regime $N_P = N_R = N$

Finally, it remains to describe the static equilibrium when we have a situation where both types of consumers purchase all differentiated goods that are available on the market, the regime $N_P = N_R = N$. There is one crucial difference to the former two cases: until now we had a situation such that the purchased good with least priority, N_i had a price that was equal to consumer i's willingness to pay for that good, $q_i(N_i)$. Now, as N_i is identical for both types of consumers, the good that has least priority for the *rich*, is priced at the willingness to pay for the *poor*. As the rich have a willingness to pay for good N that exceed its price p, the rich spend relatively more of their budget on the homogeneous good.

The system becomes easier than in the former two cases as we have now a situation where all consumers buy all goods, so $n_P = n_R = 1$. Compared to the previous regime ($N_P < N_R < N$) we now get rid of *two* variables, but have only one additional variable: the price charged by the most recent innovator p. It is clear from Lemma 1 and equation (3) that[9]

$$x_P = \frac{\nu N}{1-\gamma} \frac{p}{wb_x}. \tag{18}$$

It is not completely straightforward to find the corresponding condition for x_R. The reason is that in condition (3) the willingness to pay for the rich is strictly larger than p. Hence we cannot directly replace μ_R. To find an expression for x_R we use the budget constraints, respectively, for the rich and the poor consumers

$$\begin{aligned} wl_P + (\rho - (1-\gamma)\, g)\frac{V_{tP}}{N_t} &= p\frac{1+\nu}{1-\gamma}, \text{ and} \\ wl_R + (\rho - (1-\gamma)\, g)\frac{V_{tR}}{N_t} &= \frac{p}{1-\gamma} + wb_x x_R. \end{aligned}$$

Dividing the former equation by the latter and using our assumption on distribution allows us express x_R as

$$x_R = \frac{p}{wb_x\,(1-\gamma)} \frac{\nu\,(1-\beta\vartheta) + 1 - \vartheta}{(1-\beta)\vartheta}. \tag{19}$$

The resource constraint in regime $N_P = N_R = N$ is given by $1 = b + b_x\,[x_P + (1-\beta)x_R] + gF$. Replacing x_R and x_P by equations (18) and (19), and using $wb = 1$ allows us to rewrite the

[9] Equation (18) looks similar to the rich consumers' consumption of the traditional good (9) in the $N_P < N_R = N$ regime. However, in that regime, the most recent good with price p is sold to the rich only.

resource constraint in terms of the endogenous variables g and p

$$1 = gF + b + bp\frac{1 + v - \vartheta}{(1-\gamma)\,\vartheta}. \tag{20}$$

To get the second equation in g and p, consider the R&D equilibrium condition. The value of an innovation in the regime $N_P = N_R = N$ can be easily calculated. The innovator of date t has demand 1 from date t onwards and charges a price p that increases at rate $g\gamma$. Hence the R&D equilibrium condition can be written as

$$\frac{F}{b} = p\frac{1}{\rho} - \frac{1}{\rho + g\gamma}. \tag{21}$$

7 A graphical representation of the equilibrium

Our analysis so far has shown that the three regimes correspond to particular values of the endogenous variable n_P, the fraction of differentiated products purchased by the poor. Once we know the equilibrium value of n_P we can infer the equilibrium value of n_R, the fraction of differentiated products purchased by the rich. When $n_P \in (0, m)$, where m is defined by (15) we are in regime $N_P < N_R < N$; when $n_P \in [m, 1)$, we are in regime $N_P < N_R = N$; and trivially, when $n_P = 1$ we are in regime $N_P = N_R = N$.

To characterize the general equilibrium in a comprehensive way, it turns out convenient to work with a graphical exposition (Figure 2). The first two regimes can be addressed by focusing on the endogenous variables g and n_P. For the last regime we have to focus on the endogenous variables g and p. Figure 1 draws, respectively, the resource constraint (R-curve) and the zero-profit condition of the R&D sector (Z-curve) for the various regimes. The relevant equations are, respectively, (16) and (17) for regime $N_P < N_R < N$; (11) and (12) for regime $N_P < N_R = N$; and (20) and (21) for regime $N_P = N_R = N$. The borderline m between regimes $N_P < N_R < N$ and $N_P < N_R = N$ is given by equation (15). We note that m is determined only by exogenous parameters and not affected by the growth rate. We also see that crossing the threshold m does not lead to discrete jumps in the growth rate. This can be easily seen by setting $n_P = m$ and (recall that $\varphi(m) = 1$) in the relevant expressions for the resource constraint and the zero-profit condition.

The borderline between regimes $N_P < N_R = N$ and $N_P = N_R = N$ is $n_P = 1$ (in (g, n_P) space in Figure 2) or, equivalently, $\hat{p} > 1$ (in (g, p) space), where $\hat{p} = \varphi(1)(1-\beta) + \beta$. At this borderline, the most recent innovator is indifferent between selling to the rich at price $\varphi(1)$ and selling to the whole population at price $\hat{p}$. We note from (10) that $\varphi(1)$, and hence also $\hat{p}$, is determined only by exogenous parameters and independent of the growth rate. Also here, crossing regime thresholds does not lead to discrete jumps in the growth rate, both for the resource constraints and for the zero-profit conditions. For the R-curve this can easily be seen from setting, respectively $n_P = 1$ in equation (11) and $p = \hat{p} = \varphi(1)(1-\beta) + \beta$ in equation (20) Similarly, for the Z-curve using equations (12) and (21).

Before we proof existence of a general equilibrium we discuss the shape of the R-curve and the Z-curve in Figure 2 in more detail. This is done in the following two Lemmas.

Lemma 2 ***(zero profit condition* Z*)***

a) The Z-curve is continuous and monotonically decreasing in the (g, n_P)-space and in the (g, p)-space

b) The zero profit condition crosses the n_P-axis at n_P^Z where $p = 1 + \frac{1}{1-\beta}\frac{F\rho}{b}$ given that $1 + \frac{1}{1-\beta}\frac{F\rho}{b} \leq \hat{p}$.

c) If $1 + \frac{1}{1-\beta}\frac{F\rho}{b} > \hat{p}$ the zero profit condition crosses the p-axis at $p^Z = 1 + \frac{F\rho}{b}$

d) If $(1-\beta)\gamma + \beta\left(m - m^{1+\gamma}(1-\gamma)\right) \leq \frac{F\rho}{b}$ the $Z-$curve is only defined for values $n_P \geq m$ i.e. the regime $N_P < N_R < N$ is never reached. If $(1-\beta)\varphi(1) + \beta + \gamma - 1 \leq \frac{F\rho}{b}$ the $Z-$curve is only defined in the (g, p)-space, i.e. only the $N_P = N_R = N$ regime is compatible with positive growth rates.

Proof. see Appendix. ■

The negative slope of the $Z-$curve is intuitive. On the one side, if n_P is higher, the poors' willingness to pay $q_P(N_P)$ is higher (note that (10) implies $\partial p/\partial n_P > 0$) and the time until the poor buy Δ is shorter. Hence, the value of innovations increases in n_P. On the other side, the value of innovations increases in the growth rate g. With a higher economy-wide growth rate the willingness to pay rises faster. This is the demand effect of economic growth, and the size of this effect depends the steepness of the hierarchy. Although higher growth is also related to higher interest rates ($\partial r/\partial g = \gamma > 0$) which decreases the present value of profits, the interest rate effect is dominated by the demand effect. Taken together, the zero-profit curve has a negative slope in (g, n_P)-space. In the regime $N_P = N_R = N$ the value of an innovation is monotonically increasing in the price of the most recent innovator p, and increases in the growth rate g. Hence in this regime the zero-profit curve decreases monotonically in (g,p)-space. The continuity follows from the discussion above. At the borderline of the regimes ($n_P = m$ and $n_P = 1$, respectively) there are no discrete jumps in the growth rate.

If $(1-\beta)\,\varphi(1) + \beta + \gamma - 1 > \frac{F\rho}{b}$ (see Lemma 2d.) the $Z-$curve starts at $n_P > 0$ and $g = r = \rho/(1-\gamma)$ and then monotonically decreases. It crosses the n_P-axis or the p-axis at n_P^Z or p^Z according to the conditions in Lemma 2b and 2c. If instead $1 - \beta\varphi(1) + \beta + \gamma - 1 \leq \frac{F\rho}{b}$ (Lemma 2d.) which will be the case if the research costs F are high, the zero profit condition may only be fulfilled in the $N_P = N_R = N$ regime.

Lemma 3 ***(resource constraint*** **R*****)***

a. The RC-curve is continuous and monotonically decreasing in the (g, n_P)-space and in the (g,p)-space.

b. The resource constraint crosses the n_P-axis at $n_P^{RC} \geq m$ if $1 \geq b\,(\beta m + 1 - \beta) + \frac{\nu}{1-\gamma} b[\beta^2 m + \beta(1-\beta)m^{1-\gamma} + 1 - \beta]$.

c. The resource constraint crosses the n_P-axis at $n_P^{RC} \leq 1$ if $\left(\theta - \frac{1-\theta}{\beta}\frac{1+\nu}{\nu}\right)\frac{1-b}{b}(1-\gamma) \leq 1 + \nu - \theta$.

d. If $\left(\theta - \frac{1-\theta}{\beta}\frac{1+\nu}{\nu}\right)\frac{1-b}{b}(1-\gamma) > 1 + \nu - \theta$, the RC-curve crosses the p-axis at $p^{RC} = \frac{1-b}{b}\frac{\theta(1-\gamma)}{1+\nu-\theta}$

Proof. Part a. Follows by direct inspection of (11), (16), and (20)

Part b. The right hand side of the resource constraint (11) increases in n_P. We get the condition directly by inserting $g = 0$ into (11).

Part c. If $n_P^{RC} \leq 1$, the right hand side of (11) at $n_P = 1$ and $g = 0$ would exceed one: $1 \leq b + \frac{\nu}{1-\gamma} b\left(\beta^2 + \beta(1-\beta)\hat{p} + (1-\beta)\hat{p}\right)$. Inserting the value of $\hat{p}$ and rearranging terms, gives us the required result.

Part d. Solve (20) for p at $g = 0$. ■

Lemma 3b gives a necessary condition for existence of regime $N_p < N_R = N$. If this condition does not hold, the resource constraint can never be fulfilled in that regime and the $N_p < N_R < N$ case is the only possible equilibrium. On the other hand, the condition of Lemma 3c is necessary such that the $N_P = N_R = N$ regime is possible. Intuitively, that latter condition guarantees that the differentiated sector is sufficiently productive, such that a situation where *all* consumers buy *all* available differentiated products is feasible.

It is again instructive to discuss the reason for the negative slope of the RC-curve. In the $N_p < N_R < N$ and the $N_p < N_R = N$ regime the resource constraint (11) or (16) is falling in the (g, n_P)-space. More resources are needed when the growth rate g is higher because there are more researchers, and when the share of the products consumed by all consumers n_P is higher (note that $\varphi'(n_P) > 0$). In the $N_p < N_R < N$ regime a higher n_P also implies a higher share of products consumed by the rich $n_{R,}$ because the relative consumption of the poor m is constant. The curve starts at $g = 1/F$ and crosses the n_P-axis at n_P^{RC} (if this is smaller than unity). The Lemma 3 above exactly states the conditions on the parameter values such that $n_P^{RC} < 1$.

In the $N_p = N_R = N$ regime the resource constraint is a linear function in g and p. The first order conditions of consumer optimization imply that the expenditures on traditional goods increase if p rises, thus more resources are needed when p increases. Finally, as a higher growth rate needs more researchers, we conclude that the resource constraint is a falling line in the (g, p)-space which crosses the p-axis at p^{RC}.

Figure 2

Having discussed that shape of the zero profit conditions and the resource constraint we can turn to the problem of existence and uniqueness of the general equilibrium. To exclude the possibility that the growth rate exceeds the interest rate we assume that the resource constraint can never hold for such growth rates. Hence we take the following sufficient (but not necessary) assumption.

Assumption

$1/F < \rho/(1-\gamma)$.

We are now ready to state the following proposition.

Proposition 2 ***(existence of equilibrium).***

a. If $n_p^Z < n_P^{RC}$ or $p^Z < p^{RC}$, there exists a equilibrium with a positive growth rate.

b. If $n_p^Z \geq n_P^{RC}$ or $p^Z \geq p^{RC}$, stagnatory equilibria may arise.

7.1 Symmetry ($\gamma = 0$)

It interesting to consider the special case where the differentiated goods enter the utility function symmetrically ($\gamma = 0$). The zero profit constraint simplifies significantly because the present value of profits does not depend on the growth rate any more: The demand effect through growth is absent and the interest rate equals ρ, the exogenous rate of time preference. Hence, the $Z-$curve is a vertical line in the (g, n_P)-space and in the (g, p)-space, respectively. Thus, it is determined by the zero profit constraint alone which regime will prevail in a positive growth equilibrium. Since the $RC-$curve is still monotonically falling, the equilibrium must necessarily be *unique.* If $n_p^Z \geq n_P^{RC}$ or $p^Z \geq p^{RC}$ the unique equilibrium is stagnation.

8 The impact of inequality on growth

We have developed a model which allows us to discuss the effects inequality has on the demand structure and, in particular, on the demand for innovators. Thus, it is natural to ask how the growth rate g, the relative consumption level of the poors n_P affected if the inequality parameters β and θ change.

8.1 No traditional sector $(\nu = 0)$

To gain intuition, it is instructive to look at the baseline case where $\nu = 0$, i.e. no traditional sector exists. From Lemma 1b we know that in this case the regime $N_P = N_R = N$ can not exist (see Lemma 1b). We are able to state the following proposition

Proposition 3 *If* $\nu = 0$ *the growth rate* g *increases and the share of the poor* n_P/n_R *decreases if* θ *decreases or* β *increases.*

Proof. see Appendix. ■

The proposition states that increases in inequality in the Lorenz-sense (as captured by an increase in θ or a decrease in β) unambiguously increase growth. The intuition can be grasped by looking either at the allocation of labor or at the resulting incentives for innovations.

With a higher θ the poor become relatively richer, thus their consumption share increases, but this needs more labor in final good production what means that less researchers can be employed, this reduces growth. On the other hand, if β increases there are less people in the economy who consume all goods, hence more labor is left for research and growth rises. To get further intuition it is instructive to consider the innovation incentives. Note that the research expenditures equal the present value of the latest firms' profits in this economy. Since higher inequality rises growth, as is suggested by the proposition, it is equivalent to say that the profit share increases with inequality. But this simply means that the average markups are higher in this economy. With a less equal distribution, monopolists will charge higher markups from the rich and more product are sold at higher markups (since the consumption of the poor falls). The lower markups on products which both buy cannot dominate the first two effects.

8.2 The general case $\nu > 0$

With $\nu > 0$, we have to refer to simulations. However, we can draw general conclusions for the regime $N_P = N_R = N$, which is now possible to arise in equilibrium.

Proposition 4 *In the $N_P = N_R = N$ regime a rise in θ, i.e. decreasing inequality, unambiguously increases growth. A change in β leaves the growth rate unaffected.*

Proof. The equilibrium is characterized by equations (20) and (21). A rise in θ decreases the right hand side of (20), hence higher growth for given p is feasible (RC-curve shifts up). The parameter β does not appear in (20) and (21). ■

Since the monopolists always sell to both groups, the prices they can set are determined only by the marginal willingness to pay of the poor, as θ increases, the poor want to pay more what allows the innovators to raise prices and increase profits. These increases in innovation incentives raise the growth rate. On the other hand, a change in the groups size β can have no effect on the growth. With θ held constant the marginal willingness to pay of the relevant consumers is unaffected and innovation incentives are unchanged.

The simulations carried out in the $N_p < N_R = N$ regime render the following findings. As can be seen from Figure 3 below, increases in θ (decreasing inequality) always decreases growth, i.e. the result from the $\nu = 0$ case still holds. The figure shows further that for $\theta > 0.926$ decreasing inequality raises the growth rate, because for $\theta > 0.926$ the $N_P = N_R = N$ regime arises in equilibrium.

Figure 3

Instead, for parameter constellations where ν is high and inequality is low (θ high), we found that an increase in β (which increases inequality since θ is fixed) decreases the growth rate. An example is plotted in the second graph of Figure 3. This is an important result:

"higher inequality" *per se* is a too crude statement to judge whether and how the demand structure is affected. Loosely speaking, changes in θ affect the willingness to pay, whereas changes in β affect the market size. What is the exact mechanism which leads to the different effects of higher inequality induced by a higher β or a lower θ, respectively? Similarly to lower θ, a higher β increases inequality, but this increase will be small when θ is high.[10] This increases growth due to the implied higher profit share described above. But unlike to a lower θ, a higher β directly increases the price distortion in the monopolistic sector because the products' prices which only the rich buy must be higher as their market size $1-\beta$ is smaller than before. The increased price distortion shifts demand away from the innovative sector to the traditional x-sector, which decreases growth. The size of the latter effect is determined by the value of ν, i.e. it will dominate the first effect when ν and θ are high which confirms the results of the simulations.

9 Discussion

Our model has emphasized the role of income distribution for the incentive to conduct industrial R&D when innovations are "demand-induced". We have started out from a number of simplifying assumptions and we now discuss how robust our assumptions are with respect to these assumptions.

Assumptions on preferences We have assumed a very simple form of preferences. Goods are indivisible, either consumed or not, and there is a one-to-one mapping form wants into goods. Utility is additively separable and the consumption hierarchy can be represented by weighting the utilities from satisfying the various wants with a power function. The additive separability is essential for tractability. The restriction of the weighting function to take the power form $i^{-\gamma}$ is also essential. It implies that demand functions (and monopoly prices) only depend on the relative (rather than the absolute) position of the product in the hierarchy. As a result, the maximized static utility function can be expressed as a function of total (current) expenditure levels, the function taking the constant elasticity form with parameter γ. In other

[10] Since θ is fixed, the relative income of the rich $\frac{1-\beta\theta}{1-\beta}$ must rise since their group size has become smaller. In addition, the rise in inequality will be more substantial, the lower θ is: $\frac{\partial}{\partial\beta}\frac{1-\beta\theta}{1-\beta} = \frac{1-\theta}{(1-\beta)^2}$. Obviously, this is larger when θ is smaller.

words, in intertemporal problems with a continuum of goods, assuming additive separability and weighting by a power function is the *equivalent* of assuming a CRRA-felicity function in the one-good growth model.[11] In either case, these functional forms guarantee a constant rate of consumption growth when rates of interest and time preference are constant over time.

The assumption that goods have to be indivisible and either consumed or not, however, is not essential. As mentioned in footnote 11, every subutility function $v(\cdot)$ would do. The model could still be solved with utility functions that allow consumers to choose not only whether or not to consume a certain item, but also how much to consumeTo get the situation where poor consumers cannot afford to purchase certain items and non-negativity constraints become binding we need a subutility function with the additional property $v'(0) < \infty$. This implies that the subutility function must be non-homothetic: for instance, a quadratic form of $v(\cdot)$ has this property whereas the homothetic CES-utility function does not. , our basic results would remain unchanged.

The intertemporal utility function (1) takes the values of the static utility aggregator $u(x, \{c(j)\})$ as perfect substitutes. Hence, the implied intertemporal elasticity of substitution (IES) depends on the hierarchy parameter only and equals $1/\gamma > 1$. Consequently, the interest rate is only slowly increasing in the growth rate g: $r = \rho + g\gamma$. However, we could assume that the intertemporal utility is CRRA, i.e. it is linear in $u(x, \{c(j)\})^{1-\sigma}/(1-\sigma)$. Then, the interest rate equals $r = \rho + g(\gamma + (1-\gamma)\sigma)$. Since the interest rate is constant along a balanced growth path, the equilibrium behavior of our model remains unchanged. We can even derive stronger results: If σ is sufficiently high, it can be shown that a larger growth rate decreases the present value of profits due to higher interest rates. In that case, the Z-curve will bend rightward which implies that the equilibrium is always unique.

Differences in technologies Our model has abstracted from any differences in technologies across the various products. We made this assumption to highlight innovation incentives that are demand-induced. Nevertheless a lot of heterogeneity in the supply side can be introduced without changing the basic results. This assumption is not required in a strict sense. For instance, we could allow for different productivity levels of the various goods. This would imply

[11]More precisely, Foellmi (2003) has shown that a felicity function of the form $u(\{c(i)\}) = \int_0^\infty i^{-\gamma} v(c(i))di$ where $v' > 0 > v''$- is CRRA in expenditure levels if the price of good i can be expressed as a function of its relative position i/N only. This clearly holds true in the present model.

that the order in which the various goods are introduced may not strictly follow the hierarchy of needs.[12] We could also allow for (unanticipated) differences in the scope for technical progress for the various goods. For instance, assume, there is uncertainty with respect to technical progress at the date when a new product is introduced. Each good starts out with the "state-of-the-art" technology. With probability x a new sector is "dynamic" (costs fall with the number of previous innovations $N(t)$, just like before) and "stagnant" with probability $1-x$ (no change in costs of production). This implies that, at each date, there co-exist dynamic sectors with "state-of-the-art" productivity levels $b/N(t)$ and stagnant sectors with $b/N(s)$ where s denotes the period when the product is introduced. (The latter assumption says that *all* new sectors start out with state-of-the-art productivity but only dynamic sectors experience productivity growth). This would implies that stagnant products will disappear from the market because sooner or later costs will become larger than the prohibitive price. Some wants will not be satisfied because there are substitution possibilities with other wants (that can be satisfied with cheaper products).

Distribution of endowments Our analysis was based on the assumption that there are only two types of consumers. Extending the analysis to many different types can be easily done. A regressive transfer that involves only consumers that are rich enough to purchase all products would have no impact on incentives to innovate. Neither the size of the market nor the prices for new products would be affected as long as the "critical" consumer – the one who is indifferent between purchasing and not purchasing the good of the most recent innovator – is not affected from the redistribution. A regressive transfer between consumers that can and those that cannot afford the good of the most recent innovator would *reduce* growth. The reason is that more income for a very rich consumer – who can afford the good of the most recent innovator – does neither increase the prices nor the market size of the most recent innovator. However, when a poorer consumers has less income he will not be able to consume the new goods as soon as he would otherwise. As a result, the value of an innovation decreases. Finally, a progressive transfer between consumers both of whom cannot afford the most innovative products would increase growth. The reason is that the profit flow become more backloaded in the sense that profits accrues later in the product cycle. Discounting

[12]Murata (2003) explores the issue of technological feasibility versus desirability in the context of a static representative agent model.

implies that this reduces the profit flow and hence the value of an innovation.

A further assumption was that rich and poor consumers have the same income composition. This assumption is not essential but greatly simplifies the analysis. The assumption ensures that the distribution of income is truly exogenous. With differences in the income composition across consumers, the *personal* distribution of income and the *functional* distribution are jointly determined. The innovators' pricing decisions determine profits and the functional distribution – on the other hand, the functional distribution determines the personal distribution of income according to the (exogenously given) endowments with labor and firm shares of the households. While this analysis is much more complicated it is unlikely that this would strongly affect the results. For a preliminary analysis of this point in the context of a static model see Zehnder (2004).

Sources of technical progress Our analysis has assumed that growth is driven by the increase in technological knowledge that arises with the introduction of new final goods. It is important to keep in mind that other sources of technical progress that are potentially important were ruled out by assumption in our analysis. For instance we have assumed that no learning within sectors takes place. When the amount of learning depends on the size of the market, we would have most learning when all demand is concentrated among a few number of sectors. Clearly this would establish a bias towards a situation where inequality is harmful for growth. Similarly, incentives for the adoption of cost-saving technologies ("process innovations") are highest when markets are very large. Again, this would suggest that less inequality is favorable for technical progress as demand is spread out over a lower range of sectors.

10 Conclusions

In this paper we have used Schmookler's concept of "demand-induced invention" to study the relationship between inequality and economic growth. A natural implication of this concept is that incentives to innovate depend on the distribution of income. This is because richer consumers can afford to satisfy more wants than poorer consumers. We have shown that inequality in the distribution of incomes affects the structure of prices and the market sizes of the various products. We have also shown that poorer consumers may be excluded from certain markets (even though their willingness to pay is above the marginal cost of production) because monopolistic producers make higher profits by selling only to rich consumers at high prices.

The central result of our analysis is that the relationship between inequality and growth is not clear and depends on the nature of inequality. Higher inequality, given the group size of rich and poor and hence given a constant size of the market, tends to increase growth. This is because the most recent innovator – who sells only to rich consumers – can charge higher prices. This fosters the incentive to innovate. When higher inequality arises from a larger group of poor people – there is a higher concentration of income among a smaller number of rich people – growth may be reduced. The reason is that the market for innovators become too thin so that the profitability of an innovation decreases.

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APPENDIX

Proof of Lemma 2

a. To determine the slope of the Z-curve, it suffices to check the signs of the partial derivatives with respect to the endogenous variables. First, the value of innovations increases in n_P, in the $N_p < N_R = N$ regime, as can be seen by direct inspection of (12). Second, the value of innovations increases in the growth rate g. We derivate the value of an innovation (call it B) in the $N_P < N_R = N$ regime (7) with respect to g and we get

$$\begin{aligned}\frac{\partial B}{\partial g} &= \gamma \int_t^{t+\Delta} (1-\beta)\, pe^{-r(s-t)}(s-t)ds \\ &\quad + \gamma \int_{t+\Delta}^{\infty} \left[\beta n_P^{\gamma} + (1-\beta)p\right] e^{g\gamma(s-t)} e^{-r(s-t)}(s-t)ds\end{aligned}$$

where we used $\Pi_R(N(t+\Delta)) = \Pi_{tot}(N(t+\Delta))$. Obviously, $\partial B/\partial g > 0$.

For the $N_P = N_R = N$ regime the derivative of the value of innovation with respect to g reads

$$\frac{\partial B}{\partial g} = \gamma \int_t^{\infty} pe^{-r(s-t)}(s-t)ds > 0$$

b. To calculate the value of p where Z crosses the horizontal axis, we have to solve the zero profit condition for p where $g = 0$.

In the $N_P < N_R = N$ regime we get, using formula (7) and noting that $\Delta \to \infty$ as $g \to 0$.

$$B|_{g=0} = \int_t^{\infty} (1-\beta)(p-1)\, e^{-\rho(s-t)}ds = \frac{1-\beta}{\rho}(p-1) = \frac{F}{b}$$

We solve for p and Lemma 2b. follows immediately.

c. In the $N_P = N_R = N$ regime we use (21) and get

$$\frac{1}{\rho}\left(p^Z - 1\right) = \frac{F}{b}$$

Solving again for p^Z yields Lemma 2c.

d. The growth rate must not exceed the interest rate such that individual wealth is bounded. As the present value of profits B increases in the growth rate, we receive an upper bound for B if we insert $g = r = \rho/(1-\gamma)$. We get for the case where $n_P \geq m$ and $n_P < m$, respectively

$$\begin{aligned} B|_{g=\rho/(1-\gamma)} &= \frac{1-\beta}{\rho}\left(\varphi(n_P) - (1-\gamma)\right) + \frac{\beta}{\rho}\left(n_P - \beta n_P^{1+\gamma}(1-\gamma)\right) \quad \text{if } n_P \geq m \\ B|_{g=\rho/(1-\gamma)} &= \frac{1-\beta}{\rho}\frac{n_P}{m}\left((1-\beta)\,\gamma + \beta\left(m - m^{1+\gamma}\,(1-\gamma)\right)\right) \quad \text{if } n_P < m. \end{aligned}$$

Evaluating these expressions at $n_P = 1$ or $n_P = m$ respectively and comparing with the research cost F/b, renders us the condition in the Lemma.

Proof of Proposition 3 Consider the $N_P < N_R = N$ regime. The static equilibrium condition (10) reads, when $\nu = 0$

$$p = \frac{\beta}{1-\beta}\frac{(1-\theta)\,n_P}{\theta - n_P^{1-\gamma}} = \varphi(n_P).$$

This implies directly that $\frac{\partial\varphi}{\partial\theta} < 0$ and $\frac{\partial\varphi}{\partial\beta} > 0$.

How are the equilibrium curves defined by (11) and (12) or (20) and (21), respectively, affected? A rise in θ does not affect RC, since this parameter does not appear if $\nu = 0$. A rise in β, however, implies that less resources are needed, RC shifts up. To discuss the shifts of Z note that $\Pi_{tot}(j) = \left[\beta n_P^{\gamma} + (1-\beta)\varphi(n_P)\right]\left(\frac{j}{N}\right)^{-\gamma} - 1$ and $\Pi_R(j) = \left[\varphi(n_P)\left(\frac{j}{N}\right)^{-\gamma} - 1\right](1-\beta) = \varphi(n_P)(1-\beta)\left(\frac{j}{N}\right)^{-\gamma} + \beta$. Using the formula for $\varphi(n_P)$ above we get the expression $\varphi(n_P)(1-\beta) = \beta\frac{(1-\theta)n_P}{\theta - n_P^{1-\gamma}}$. Hence, $\varphi(n_P)(1-\beta)$ falls in θ and increases in β. With n_P fixed, we directly get the result that $\frac{\partial\Pi_{tot}(j)}{\partial\theta} < 0$, $\frac{\partial\Pi_R(j)}{\partial\theta} < 0$ and $\frac{\partial\Pi_{tot}(j)}{\partial\beta} > 0$, $\frac{\partial\Pi_{tot}(j)}{\partial\beta} > 0$. Consequently, the Z-curve shifts to the right when θ increases and it shifts to left when β increases.

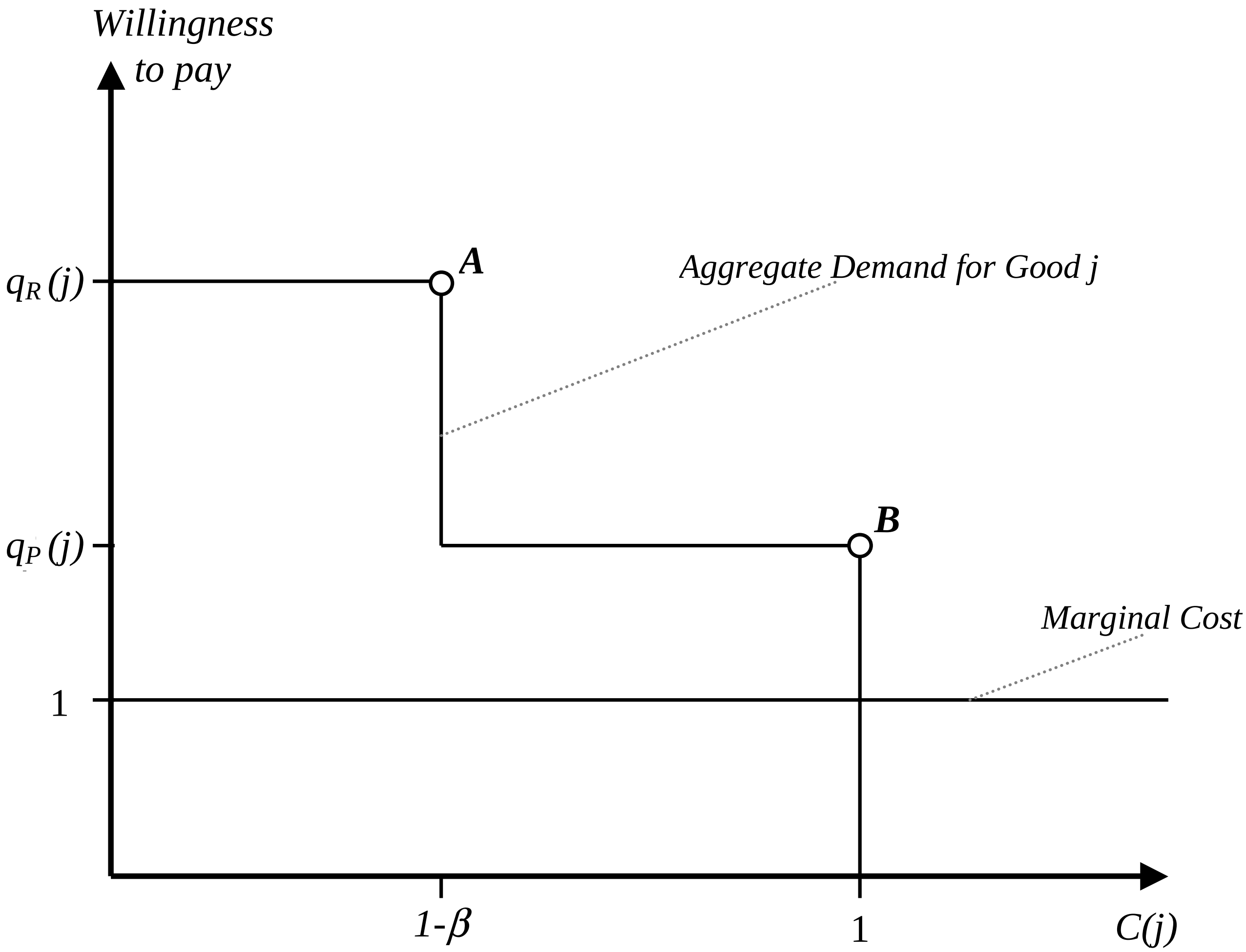


Figure 1: Aggregate Demand for good j and Decision Problem of the Monopolist

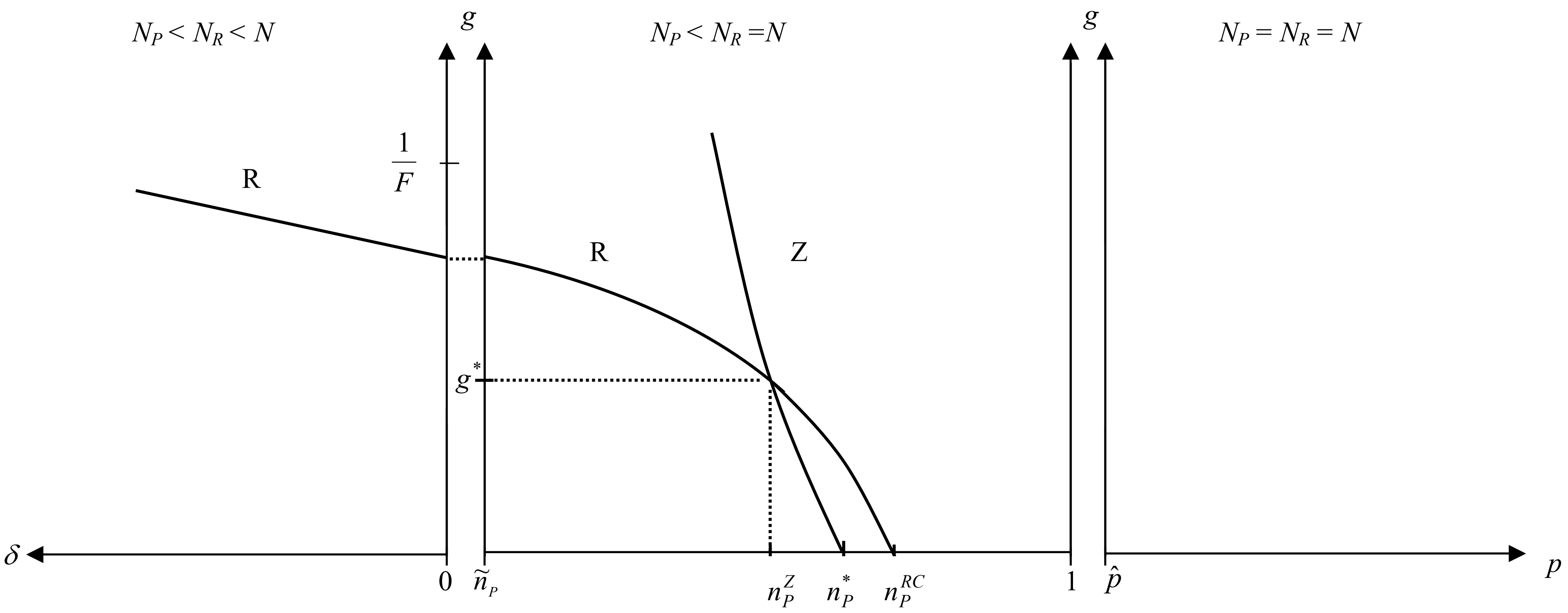


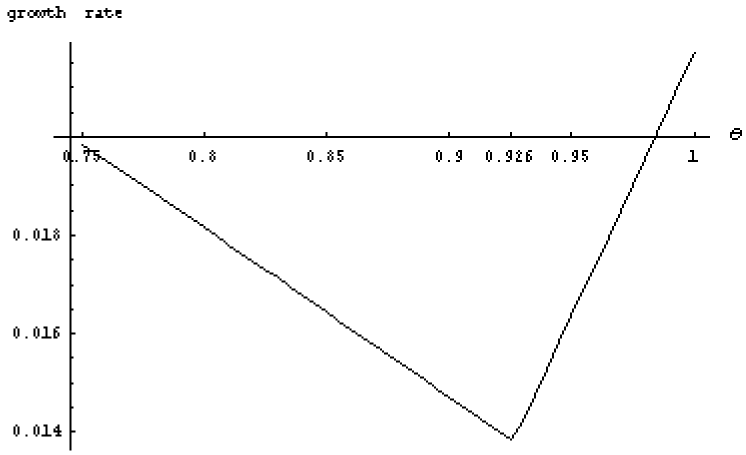
Figure 2: A Unique Positive Growth Equilibrium

Figure 3: Simulations

Default values:
$\theta = 0.8$, $\beta = 0.5$, $F = 5$, $b = 0.3$, $\sigma = 2$, $\rho = 0.02$, $\gamma = 0.3$, $\nu = 0.8$

The growth rate in dependence of θ

The regime switch, where $n_P = 1$, arises at $\theta = 0.926$



The growth rate in dependence of β